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A MODEL OF LIQUID FLOW AND INJECTION IN  
A REGENERATIVE LIQUID PROPELLANT GUN

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DECEMBER 1989

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## I. INTRODUCTION

A number of interior ballistic models have been developed for the regenerative liquid propellant gun, e.g. the models developed by Gough<sup>1,2</sup> and Coffee.<sup>3</sup> These and similar models are capable of accurately simulating the overall performance of the regenerative liquid propellant gun (RLPG). However, some portions of the interior ballistic process are not treated in detail in these models. In particular, the liquid injection process is usually modeled using the steady state Bernoulli equation with a correlation to account for the pressure drop across the orifice.

The authors have previously presented a model of the liquid injection process in a regenerative liquid propellant gun.<sup>4-6</sup> This model was developed to investigate the unexpectedly high values of the liquid discharge coefficient observed in regenerative gun firings.<sup>7,8</sup> The model is based on an adaptation of the Lagrange approximation to the liquid flow in the reservoir of an RLPG.

In this paper, a modified development of the model equations is presented. The primary modification is the inclusion of the full pressure distribution over the entire reservoir in the calculation of the space mean pressure, and subsequently in the model equations. In the earlier version of the model,<sup>4</sup> the injection orifice and the region near the head of the injection piston were excluded in the calculation of the space mean pressure. The velocity distribution in the liquid has also been modified, eliminating another approximation used in the earlier version of the model.

The resulting model equations are then applied to a simplified RLPG geometry, in order to evaluate the impact of utilizing a Lagrange gradient model in the

propellant reservoir. In a companion report,<sup>9</sup> the model is applied to the simulation of the RLPG interior ballistic process, and comparisons with experimental data are presented.

## II. DEFINITION OF THE CONTROL VOLUME

The liquid injection model, as currently formulated, is applicable to the regenerative gun configurations known as Concept VI and Concept VIA, shown in Figures 1 and 2 respectively. The interior ballistic process is initiated by firing an igniter which pressurizes the combustion chamber. The chamber pressure acting on the injection piston forces it to the rear, compressing the liquid in the reservoir. After an initial transient period, the pressure in the liquid reservoir will exceed the combustion chamber pressure as a result of the differential area across the injection piston. As the injection piston moves to the rear, opening the injection orifice, liquid propellant will be injected into the combustion chamber.

Concepts VI and VIA are very similar in design. The primary differences are the elimination of the contour at the rear of the control rod in Concept VI (which is used to decelerate the piston at the end of its travel) and the introduction of a damping fluid at the rear of the injection piston shaft to control its motion in Concept VIA. In the development of the model, we will focus on Concept VI, but the simplifications in applying the model to Concept VIA are straightforward.<sup>3</sup>

The control volume for Concept VI is illustrated in Figure 3. The contours of the piston and the reservoir are approximated by straight line segments as indicated. The control rod and transducer block are fixed in the reference frame of the chamber. The origin of the coordinate system fixed in the chamber frame

of reference is at the rear (left hand) end of the reservoir, and  $x$  is the coordinate along the control rod. The piston moves rearward with a velocity  $u_p(t)$ , and the points  $s_1(t)$ ,  $s_2(t)$ , and  $s_3(t)$  are the coordinates of the fixed stations on the inner contour of the piston in the  $x$  coordinated system.

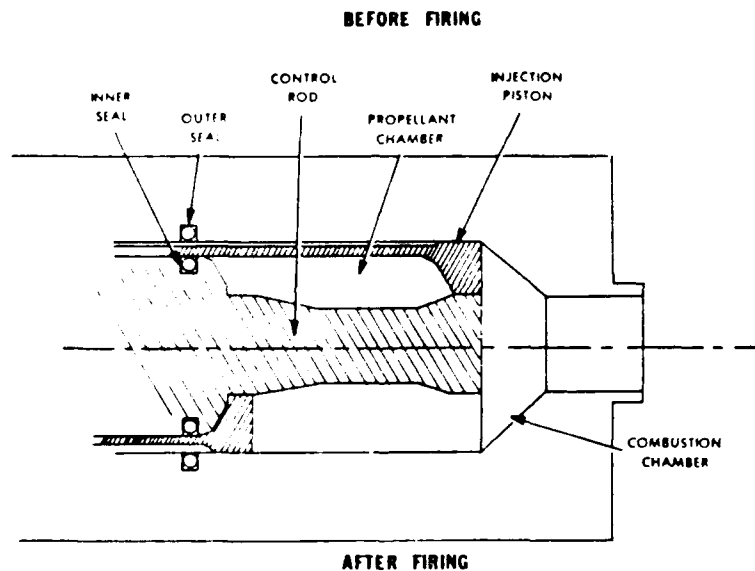


Figure 1. Concept VI. Regenerative Liquid Propellant Gun.

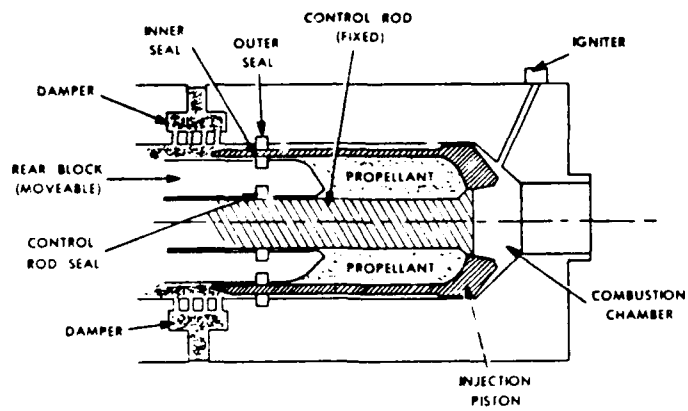


Figure 2. Concept VIA. Regenerative Liquid Propellant Gun.

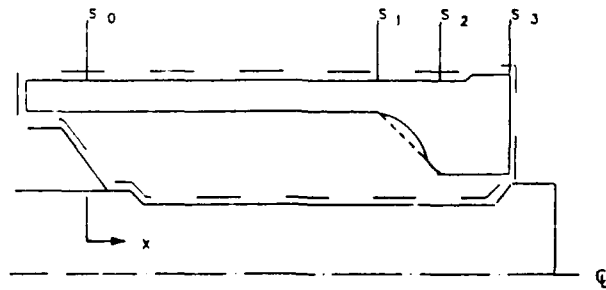


Figure 3. Control Volume for Concept VI. RLPG.

Consider a fixed station  $x$  along the control rod. At  $t = 0$ , the corresponding point on the inner contour of the injection piston is defined as  $s(x,0)$ . At the some later time  $t$ , the piston has moved to the rear and the point on the inner contour of the injection piston at the fixed station  $x$  is now

$$s(x,t) = s(x,0) + \int_0^t u_p(t') dt' \quad (1)$$

such that,

$$\frac{\partial s}{\partial t} \Big|_x = u_p \quad (2)$$

and

$$\frac{\partial s}{\partial x} \Big|_t = 1 \quad (3)$$

Note that the right hand face of the control volume coincides with the exit plane of the injection orifice,  $s_3(t)$ , such that the control volume also varies with time.

The cross sectional area of the control volume at a point  $x$  along the control rod is

$$A(x,t) = \pi[R^2(x,t) - r_b^2(x)] \quad (4)$$

where  $R(x,t)$  is the inner radius of the injection piston at the point  $x$  at time  $t$ , and  $r_b(x)$  is the radius of the control rod at the point  $x$ . The volume of the liquid is then defined by

$$V_R(t) = \int_0^{s_3(t)} A(x,t) dx \quad (5)$$

and it can be shown that the rate at which the control volume changes is

$$\dot{V}_R(t) = -u_p[A_R + A_3(t)] = -u_p A_L(t) \quad (6)$$

where  $A_R$  is the cross sectional area of the reservoir side of the injection piston,  $A_3(t)$  is the area of the injection orifice at the point  $s_3(t)$ , and  $A_L(t) = A_R + A_3(t)$ .

### III. LAGRANGE APPROXIMATION APPLIED TO THE RESERVOIR

The equations of motion for the liquid in the reservoir, continuity and momentum equations, are written to include area change as the piston moves rearward. We note that the area through which the fluid flows is a function of both time and position, since the contoured piston moves rearward over a contoured bolt. The one-dimensional equations of motion are then

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho u A)}{\partial x} = 0 \quad (7)$$

and,

$$\frac{\partial(\rho u A)}{\partial t} + \frac{\partial(\rho u^2 A)}{\partial x} = -A \frac{\partial P}{\partial x} \quad (8)$$

where  $\rho$ ,  $u$ ,  $A$  and  $P$  are all functions of both position and time.

The Lagrange assumption, density is a function of time only and is thus constant over the control volume such that the spatial derivative is zero, is

a good approximation in the case of the LP reservoir since the liquid density only varies by about 4% over the entire ballistic cycle and the spatial variation over the reservoir at any given time is much less than this. Therefore, applying the Lagrange approximation, Equation (7) becomes

$$A \frac{1}{\rho} \frac{\partial \rho}{\partial t} = - \frac{\partial A}{\partial t} - \frac{\partial u A}{\partial x} \quad (9)$$

Noting that  $\rho = m_L / V_R$ , where  $m_L$  is the mass of liquid remaining in the reservoir at time  $t$  and

$$m_L = - \rho A_3 (u_3 + u_p) \quad (10)$$

is the mass flux out of the control volume, and using Equation (6) we obtain,

$$\frac{1}{\rho} \frac{\partial \rho}{\partial t} = - \frac{(u_3 A_3 - u_p A_R)}{V_R} \quad (11)$$

The numerator on the right hand side of Equation (11) is just the difference between the volume of liquid exiting the reservoir per unit time and the volume swept out per unit time by the piston motion,  $[(u_3 + u_p)A_3 - u_p(A_3 + A_R)]$ . If the liquid were assumed to be incompressible, these terms would be identical by definition and  $\partial \rho / \partial t = 0$ .

The time derivative of the area,  $A(x,t)$ , is evaluated using Equations (2) and (4), giving us

$$\frac{\partial A(x,t)}{\partial t} = \frac{\partial A}{\partial s} \frac{\partial s}{\partial t} \Big|_x = u_p \frac{\partial A}{\partial s} \Big|_x \quad (12)$$

or,

$$\frac{\partial A(x,t)}{\partial t} = -u_p \frac{\partial[\pi R^2(x,t)]}{\partial x} \quad (13)$$

Using Equations (10) and (12) in Equation (9) we obtain,

$$\frac{\partial v A}{\partial x} = -u_p \frac{\partial[\pi R^2(x,t)]}{\partial x} + \left\{ \frac{v_3 A_3 - u_p A_R}{V_R} \right\} A(x,t) \quad (14)$$

and upon integrating we have

$$v(x,t) A(x,t) = u_p \{ \pi[R_0^2 - R^2(x,t)] \} + (v_3 A_3 - u_p A_R) \frac{V(x,t)}{V_R} \quad (15)$$

where  $R_0$  is the inner radius of the injection piston at the origin, i.e. the rear wall of the reservoir.

Rewriting Equation (15) we have

$$v(x,t) A(x,t) = u_p a(x,t) + (v_3 A_3 - u_p A_R) \frac{V(x,t)}{V_R} \quad (16)$$

where,

$$a(x,t) = \pi R_0^2 - \pi R^2(x,t) \quad (17)$$

such that

$$A(x,t) + a(x,t) = [\pi R_0^2 - \pi r_0^2(x)] \quad (18)$$

This velocity distribution, Equation (16), exhibits the correct behavior at the boundaries of the control volume,

$$v(0,t) = 0 \quad (19)$$



and

$$v(s_3, t) = v_3 \quad (20)$$

where we have used the fact that,

$$a(s_3, t) = A_R \quad (21)$$

Using Equation (7), the momentum equation, (8), can be rewritten in the form

$$\frac{\partial P}{\partial x} = - \rho v \frac{\partial v}{\partial x} - \rho \frac{\partial v}{\partial t} \quad (22)$$

Integrating on  $[0, x]$  we have

$$P(x, t) = P_0(t) - \frac{1}{2} \rho v^2(x, t) - \rho \int_0^x v(x', t) dx' \quad (23)$$

which is a form of the unsteady Bernoulli equation. Evaluating Equation (23) at  $s_3$ , we have

$$P_3(t) = P_0(t) - \frac{1}{2} \rho v_3^2 - \rho \int_0^{s_3} v(x, t) dx \quad (24)$$

also a form of the unsteady Bernoulli equation, where  $P_3(t)$  is defined as the pressure at the right hand boundary of the control volume, and is just the gas pressure in the combustion chamber.

The space mean pressure in the liquid reservoir is then defined by

$$\bar{P}(t) \int_0^{s_3} A(x, t) dx = \int_0^{s_3} P(x, t) A(x, t) dx \quad (25)$$

or

$$\bar{P}(t) = \frac{1}{V_R} \int_0^{s_3} P(x,t) A(x,t) dx \quad (26)$$

Using Equation (23), we obtain

$$\begin{aligned} P_0(t) = \bar{P}(t) &+ \frac{\rho}{2V_R} \int_0^{s_3} v^2(x,t) A(x,t) dx \\ &+ \frac{\rho}{V_R} \int_0^{s_3} A(x,t) \int_0^x v(x',t) dx' dx \end{aligned} \quad (27)$$

where  $\bar{P}(t)$  is obtained from the equation of state for liquid.

Equation (27) is utilized in Equation (23) to determine the pressure distribution in the reservoir, and in Equation (24) to complete the definition of the unsteady Bernoulli equation for the reservoir. We now consider the integrals involved in these equations.

#### 1. PRESSURE DISTRIBUTION:

In order to evaluate the integral in Equation (24) we first differentiate Equation (16) to obtain

$$\begin{aligned} v(x,t) A(x,t) + v(x,t) A(x,t) &= \dot{u}_p \alpha(x,t) + u_p \dot{\alpha}(x,t) \\ &- (\dot{u}_p A_R - v_3 A_3 - v_3 A_3) \frac{V(x,t)}{V_R} \\ &- (u_p A_R - v_3 A_3) \left[ \frac{V(x,t)}{V_R} - \frac{V_R V(x,t)}{V_R^2} \right] \end{aligned} \quad (28)$$

where

$$A(x,t) = - u_p \frac{\partial \alpha(x,t)}{\partial x} \quad (29)$$

$$\alpha(x,t) = u_p \frac{\partial \alpha(x,t)}{\partial x} \quad (30)$$

$$A_3(t) = u_p \frac{\partial \pi r_b^2(x)}{\partial x} \Big|_{s_3} \quad (31)$$

$$V(x,t) = - u_p \alpha(x,t) \quad (32)$$

and

$$V_R(t) = - u_p A_L(t) \quad (33)$$

Substituting Equations (29) through (33) in Equation (28) and rearranging terms, we obtain

$$\begin{aligned} v(x,t) = & \left\{ u_p^2 \left( \frac{\alpha(x,t)}{A^2(x,t)} - \frac{\partial \alpha(x,t)}{\partial x} \right) - u_p \left( u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right) \left( \frac{V(x,t)}{V_R} \frac{A_L}{A^2(x,t)} \frac{\partial \alpha(x,t)}{\partial x} \right) \right\} \\ & + \left\{ u_p \left( \frac{\alpha(x,t)}{A(x,t)} \right) + u_p^2 \left( \frac{1}{A(x,t)} - \frac{\partial \alpha(x,t)}{\partial x} \right) \right\} \\ & - \left[ u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right] \left( \frac{A_L}{A(x,t)} \frac{V(x,t)}{V_R} \right) \\ & + u_p v_3 \left( \frac{V(x,t)}{V_R} \frac{1}{A(x,t)} - \frac{\partial \pi r_b^2(x)}{\partial x} \Big|_{s_3} \right) \\ & + u_p \left[ u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right] \left\{ \frac{A_L}{V_R} \left[ \frac{\alpha(x,t)}{A(x,t)} - \frac{A_L}{A(x,t)} \frac{V(x,t)}{V_R} \right] \right\} \quad (34) \end{aligned}$$

Now, using Equation (34), the integral in Equation (23) becomes

$$\begin{aligned}
\int_0^x \dot{v}(x', t) dx' &= u_p^2 J_1(x, t) - u_p \left[ u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right] J_2(x, t) \\
&+ u_p L_1(x, t) + u_p^2 J_3(x, t) - \left[ u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right] L_2(x, t) \\
&+ u_p v_3 J_4(x, t) + u_p \left[ u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right] J_5(x, t) ,
\end{aligned} \tag{35}$$

where,

$$J_1(x, t) = \int_0^x \frac{a(x', t)}{A^2(x', t)} \frac{\partial a(x', t)}{\partial x'} dx' \tag{36}$$

$$J_2(x, t) = \int_0^x \frac{V(x', t)}{V_R} \frac{A_L}{A^2(x', t)} \frac{\partial a(x', t)}{\partial x'} dx' \tag{37}$$

$$J_3(x, t) = \int_0^x \frac{1}{A(x', t)} \frac{\partial a(x', t)}{\partial x'} dx' \tag{38}$$

$$J_4(x, t) = \frac{\partial \pi r_b^2(x)}{\partial x} \Big|_s \int_0^x \frac{V(x', t)}{V_R} \frac{dx'}{A(x', t)} \tag{39}$$

$$J_5(x, t) = \frac{A_L}{V_R} \int_0^x \left[ \frac{a(x', t)}{A(x', t)} - \frac{A_L}{A(x', t)} \frac{V(x', t)}{V_R} \right] dx' \tag{40}$$

$$L_1(x, t) = \int_0^x \frac{a(x', t)}{A(x', t)} dx' \tag{41}$$

and

$$L_2(x, t) = \frac{A_L}{V_R} \int_0^x \frac{V(x', t)}{A(x', t)} dx' . \tag{42}$$

Note that the integrals  $J_1$  through  $J_5$  are dimensionless, while the integrals  $L_1$  and  $L_2$  have units of length. Rearranging terms in Equation (35) and substituting in Equation (23), the pressure distribution in the liquid becomes.

$$\begin{aligned}
 P(x,t) = P_0(t) & - \frac{1}{2}\rho v^2(x,t) - \rho u_p^2 [J_1(x,t) + J_3(x,t)] \\
 & + \rho u_p \left[ u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right] [J_2(x,t) - J_5(x,t)] \\
 & - \rho u_p v_3 [J_4(x,t)] - \rho u_p \left[ L_1(x,t) - \frac{A_R}{A_L} L_2(x,t) \right] \\
 & - \rho v_3 \left[ \frac{A_3}{A_L} L_2(x,t) \right] .
 \end{aligned} \tag{43}$$

## 2. UNSTEADY BERNOULLI EQUATION:

Having evaluated the integral in Equation (23), the corresponding integral in Equation (24) can be determined in a straight forward manner. The integrals defined in Equations (36)-(42) are evaluated at  $s_3$ , and are written in the form

$$J_1(s_3, t) = J_1^{03}(t) \tag{44}$$

and

$$L_1(s_3, t) = L_1^{03}(t) \tag{45}$$

etc., such that Equation (24) becomes

$$\begin{aligned}
P_3(t) = P_0(t) &- \frac{1}{2} \rho u_3^2 - \rho u_p^2 [J_1^{03}(t) + J_3^{03}(t)] \\
&+ \rho u_p \left[ u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right] [J_2^{03}(t) - J_5^{03}(t)] \\
&- \rho u_p v_3 [J_4^{03}(t)] - \rho u_p \left[ L_1^{03}(t) - \frac{A_R}{A_L} L_2^{03}(t) \right] \\
&- \rho v_3 \left[ \frac{A_3}{A_L} L_2^{03}(t) \right].
\end{aligned} \tag{46}$$

### 3. EVALUATION OF $P_0(t)$ :

In order to determine  $P_0(t)$ , which is required to complete Equations (43) and (46), we must evaluate the two integrals on the right hand side of Equation (27). Using Equation (16), the first integral can be written in the form,

$$\begin{aligned}
\frac{\rho}{2V_R} \int_0^{s_1} u^2(x,t) A(x,t) dx &= \frac{\rho u_p^2}{2} [J_6^{03}(t)] \\
&- \rho u_p \left( u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right) [J_7^{03}(t)] \\
&+ \frac{\rho}{2} \left( u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right)^2 [J_8^{03}(t)] \\
&= \frac{\rho}{2} \overline{u^2(t)}.
\end{aligned} \tag{47}$$

where,

$$J_6^{03}(t) = \frac{1}{V_R} \int_0^{s_1} \frac{a^2(x,t)}{A(x,t)} dx \tag{48}$$

$$J_7^{03}(t) = \frac{A_L}{V_R} \int_0^{s_1} \frac{V(x,t)}{V_R} \frac{a(x,t)}{A(x,t)} dx \tag{49}$$

$$J_0^{03}(t) = \frac{A_L}{V_R} \int_0^{S_2} \left[ \frac{V(x,t)}{V_R} \right]^2 \frac{A_L}{A(x,t)} dx \quad (50)$$

The second integral in Equation (27) is easily evaluated by noting that it is simply the volume average of Equation (35) over the reservoir. Applying this observation to the integrals in Equations (36)-(42) and defining

$$\overline{J_1^{03}(t)} = \frac{1}{V_R} \int_0^{S_2} A(x,t) J_1(x,t) dx \quad (51)$$

and

$$\overline{L_1^{03}(t)} = \frac{1}{V_R} \int_0^{S_2} A(x,t) L_1(x,t) dx \quad (52)$$

etc., we have

$$\begin{aligned} \frac{\rho}{V_R} \int_0^S A(x,t) \int_0^x v(x',t) dx' dx &= \rho u_p^2 [\overline{J_1^{03}(t)} + \overline{J_3^{03}(t)}] \\ &- \rho u_p \left[ u_p \frac{A_R}{A_L} - u_3 \frac{A_3}{A_L} \right] [\overline{J_2^{03}(t)} - \overline{J_5^{03}(t)}] \\ &+ \rho u_p u_3 [\overline{J_4^{03}(t)}] + \rho u_p \left[ \overline{L_1^{03}(t)} - \frac{A_R}{A_L} \overline{L_2^{03}(t)} \right] \\ &+ \rho u_3 \left[ \frac{A_3}{A_L} \overline{L_2^{03}(t)} \right] \\ &= \rho \overline{v l(t)} \quad (53) \end{aligned}$$

The pressure at the rear wall is then determined by substituting Equations (47) and (53) in Equation (27),

$$P_0(t) = \bar{P}(t) + \frac{\rho}{2} \overline{v^2(t)} + \rho \overline{v l(t)} \quad (54)$$

#### IV. MOMENTUM EQUATION

The momentum equation for the control volume shown in Figure 3, in the reference frame of the chamber is,

$$M_p \ddot{u}_p + \frac{\partial}{\partial t} \int_{CV} \bar{v} \rho dV + \int_{CS} \bar{v} \rho \bar{v} \cdot d\bar{A} = - \int P d\bar{A} , \quad (55)$$

where  $d\bar{A}$  is the outward directed normal from the element of control surface.

Rewriting Equation (54) we have,

$$\begin{aligned} - M_p \ddot{u}_p \hat{i} + \frac{\partial}{\partial t} \left\{ \int_0^{s_1} \rho v A dx \right\} \hat{i} + \rho v_3^2 A_3 \hat{i} = \\ [P_0 A_T + P_{CF} A_s - P_3 (A_p + A_3)] \hat{i} , \end{aligned} \quad (56)$$

where the control volume has been extended to include the piston shaft;  $P_{CF}$  is the pressure exerted by a control fluid on the area of the piston shaft,  $A_s$ ;  $A_p$  is the area of the chamber face of the piston; and  $A_T$  is the total area of the transducer block, including the forward axial projected area of the damping taper on the bolt in Concept VI. The term which accounts for the control fluid is used for the Concept VIA configuration, and will be ignored for the moment. Thus, Equation (55) becomes,

$$M_p \ddot{u}_p - \frac{\partial}{\partial t} \left\{ \int_0^{s_1} \rho v A dx \right\} = P_3 (A_p + A_3) - P_0 A_T + \rho v_3^2 A_3 , \quad (57)$$

where the second term on the left-hand side is just  $\frac{d}{dt} \{m_L \bar{v}(t)\}$ .



We will first evaluate the integral in the above equation and then take the time derivative. Using Equation (16), we have

$$\begin{aligned} \rho \int_0^{s_3} \epsilon(x,t) A(x,t) dx &= \rho u_p \int_0^{s_3} a(x,t) dx \\ &- \rho(u_p A_R - v_3 A_3) \frac{1}{V_R} \int_0^{s_3} V(x,t) dx \end{aligned} \quad (58)$$

or

$$\rho \int_0^{s_3} \epsilon(x,t) A(x,t) dx = m_L u_p J_9^{03}(t) - \frac{m_L(u_p A_R - v_3 A_3)}{V_R} L_3^{03}(t) \quad (59)$$

where,

$$J_9^{03}(t) = \frac{1}{V_R} \int_0^{s_3} a(x,t) dx \quad (60)$$

$$L_3^{03}(t) = \frac{1}{V_R} \int_0^{s_3} V(x,t) dx \quad (61)$$

Now, taking the derivative, we have

$$\begin{aligned} \frac{\partial}{\partial t} \left\{ \rho \int_0^{s_3} \epsilon(x,t) A(x,t) dx \right\} &= m_L u_p \left[ J_9^{03}(t) - \frac{A_R}{V_R} L_3^{03}(t) \right] \\ &+ m_L v_3 \left[ \frac{A_3}{V_R} L_3^{03}(t) \right] + \rho u_p v_3 \left[ L_3^{03}(t) \frac{\partial \pi r_0^2(x)}{\partial x} \Big|_{s_3} \right] + \rho u_p (u_p A_R + v_3 A_3) \\ &+ 2\rho u_p (u_p A_R - v_3 A_3) J_9^{03}(t) \\ &- \frac{\rho}{V_R} (u_p A_R - v_3 A_3) [(u_p A_R - v_3 A_3) + u_p A_L] L_3^{03}(t) \end{aligned} \quad (62)$$

Substituting in Equation (57), we obtain

$$\begin{aligned}
M_p u_p \left\{ 1 - \frac{m_i}{M_p} \left[ J_9^{03}(t) - \frac{A_R}{V_R} L_3^{03}(t) \right] \right\} &= m_L v_3 \left[ \frac{A_3}{V_R} L_3^{03}(t) \right] \\
&= P_3(A_p + A_3) - P_0 A_T + \rho v_3^2 A_3 + \rho u_p (u_p A_R + v_3 A_3) \\
&+ \rho u_p v_3 \left[ L_3^{03}(t) \frac{\partial \pi r_b^2(x)}{\partial x} \Big|_{s_3} \right] + 2\rho u_p (u_p A_R - v_3 A_3) J_9^{03}(t) \\
&- \frac{\rho}{V_R} (u_p A_R - v_3 A_3) [(u_p A_R - v_3 A_3) + u_p A_L] L_3^{03}(t)
\end{aligned} \tag{63}$$

which is the force balance equation for the piston

## V. EQUATIONS OF MOTION

The unsteady Bernoulli equation, Equation (46), and the force balance equation for the piston, Equation (63), are the coupled, ordinary differential equations of motion governing liquid injection and injector piston motion. These equations can be rewritten in the form,

$$v_3 L_v^{e//}(t) - u_p L_u^{e//}(t) = \frac{1}{\rho} [\bar{P}(t) - P_3(t)] - \frac{1}{2} v_3^2 + U^2(t) \tag{64}$$

and

$$u_p M_p^{e//}(t) - v_3 m_L^{e//}(t) = P_3(t) [A_p + A_3] - \bar{P}(t) A_T + \rho v_3^2 A_3 - \rho [U^2 A(t)] \tag{65}$$

respectively, where we have used Equation (54) to eliminate  $P_0(t)$  and where

$$L_u^{e//}(t) = \frac{A_R}{A_L} L_2^{03}(t) - L_1^{03}(t) \tag{66}$$

$$L_v^{e//}(t) = \frac{A_3}{A_L} L_2^{03}(t) \tag{67}$$

$$\begin{aligned}
U^2(t) = & u_p^2 \left[ \overline{J_1^{03}(t)} - J_1^{03}(t) \right] + \left[ \overline{J_3^{03}(t)} - J_3^{03}(t) \right] + \frac{1}{2} J_6^{03}(t) \Big] \\
& + u_p \left( u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right) \left[ \left( J_2^{03}(t) - \overline{J_2^{03}(t)} \right) - \left( J_5^{03}(t) - \overline{J_5^{03}(t)} \right) - J_7^{03}(t) \right] \\
& - u_p v_3 \left[ J_4^{03}(t) - \overline{J_4^{03}(t)} \right] + \frac{1}{2} \left( u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right)^2 J_8^{03}(t) \\
& + u_p \left[ \overline{L_1^{03}(t)} - \frac{A_R}{A_L} \overline{L_2^{03}(t)} \right] + v_3 \left[ \frac{A_3}{A_L} \overline{L_2^{03}(t)} \right] \quad (68)
\end{aligned}$$

$$M_p^{eff}(t) = M_p \left\{ 1 + \frac{m_L}{M_p} \left[ \frac{A_R}{V_R} L_3^{03}(t) - J_9^{03}(t) \right] \right\} \quad (69)$$

$$m_L^{eff}(t) = m_L \left\{ \frac{A_3}{V_R} L_3^{03}(t) \right\} \quad (70)$$

$$\begin{aligned}
U^2 A(t) = & A_T \left\{ u_p^2 \left[ \overline{J_1^{03}(t)} + \overline{J_3^{03}(t)} + \frac{1}{2} J_6^{03}(t) \right] \right. \\
& - u_p \left( u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right) \left[ \overline{J_2^{03}(t)} - \overline{J_5^{03}(t)} + J_7^{03}(t) \right] \\
& + u_p v_3 \left[ \overline{J_4^{03}(t)} \right] + \frac{1}{2} \left( u_p \frac{A_R}{A_L} - v_3 \frac{A_3}{A_L} \right)^2 \left[ J_8^{03}(t) \right] \\
& + u_p \left[ \overline{L_1^{03}(t)} - \frac{A_R}{A_L} \overline{L_2^{03}(t)} \right] + v_3 \left[ \frac{A_3}{A_L} \overline{L_2^{03}(t)} \right] \Big\} \\
& - u_p (u_p A_R + v_3 A_3) - u_p v_3 \left[ L_3^{03}(t) \frac{\partial \pi r_6^2(x)}{\partial x} \Big|_{s_3} \right] \\
& - 2u_p (u_p A_R - v_3 A_3) J_9^{03}(t) \\
& + \left( \frac{u_p A_R - v_3 A_3}{V_R} \right) \left[ (u_p A_R - v_3 A_3) + u_p A_L \right] L_3^{03}(t) \quad (71)
\end{aligned}$$

The functions  $U^2(t)$  and  $U^2 A(t)$  have dimensions of velocity squared and velocity squared times area respectively, as suggested by the function names. Note that

these functions include terms involving piston and liquid acceleration,  $u_p$  and  $u_3$ , arising from the evaluation of the  $P_0(t)$  in terms of the space mean pressure  $\overline{P(t)}$ .

The model equations have been incorporated into a computer model and applied to Concept VI and Concept VIA RLPG configurations. The results of computer simulations, and comparisons with experimental data are presented in the accompanying paper.<sup>9</sup>

## VI. DISCUSSION

The model of the LP reservoir and injection process which has been developed in the preceding sections is based on the application of the Lagrange approximation to a flow system in which the cross sectional area varies both with position along the direction of flow and with time at a fixed position in the system. As a result, the velocity gradient, Equation (16), is quite different from the more usual Lagrange gradient to which we are accustomed.

Consider the development of the standard Lagrange velocity gradient from the continuity equation,

$$\frac{\partial t(x,t)}{\partial x} = \left[ -\frac{\rho}{\rho} \right] \quad (72)$$

where it is assumed that  $\rho$  is a function of time but not position along the flow.

Integrating, and assuming that  $v(0,t)=0$ , we have

$$v(x,t) = \left[ -\frac{\rho}{\rho} \right] x, \quad (73)$$

where

$$\left[ -\frac{\rho}{\rho} \right] = \frac{V}{V} - \frac{m}{m}. \quad (74)$$

For the standard Lagrange gun, in which the propellant mass is fixed and density varies only as a result of projectile motion, the term in brackets on the right-hand side of Equation (73) is just the projectile velocity divided by the projectile travel, giving us the familiar form of the linear Lagrange velocity distribution.

If we now permit the cross sectional area to vary with position along the flow system, but assume that the time derivative is zero, the continuity equation becomes

$$\frac{\partial \rho(x,t) A(x)}{\partial x} = \left[ -\frac{\rho}{\rho} \right] A(x). \quad (75)$$

Integrating, and again assuming that  $\rho(0,t)=0$ , we have

$$\rho(x,t) A(x) = \left[ -\frac{\rho}{\rho} \right] V(x). \quad (76)$$

or

$$\rho(x,t) = \left[ -\frac{\rho}{\rho} \right] \frac{V(x)}{A(x)}. \quad (77)$$

From Equation (76), we can see that the mass flux at a position,  $x$ , in the flow is proportional to the time rate of change of the overall system density, times that portion of the system volume from the origin to the point  $x$  along the flow, i.e. the mass flux varies linearly with volume along the flow. As a result, the velocity distribution is proportional to the volume to area ratio, Equation (77).

Finally, we permit the cross sectional area to vary with time at a fixed position in the flow, as well as with position along the flow. The continuity equation, (9), can be rewritten to give,

$$\frac{\partial \rho(x,t)A(x,t)}{\partial x} = \left[ -\frac{\dot{\rho}}{\rho} \right] A(x,t) - \frac{\partial A(x,t)}{\partial t}. \quad (78)$$

Integrating, and assuming that  $v(0,t)=0$ , we have

$$v(x,t)A(x,t) = \left[ -\frac{\dot{\rho}}{\rho} \right] V(x,t) - \int_0^x \frac{\partial A(x',t)}{\partial t} dx'. \quad (79)$$

The first term on the right-hand side of Equation (79) is the flux arising from changes in system volume (expansion or compression) and from mass loss or mass addition to the system as a whole. As noted previously, this flux varies linearly with volume along the flow. The second term is a flux arising from the compression of the flow due to a change in flow area. If the integral is negative, i.e. if the flow area between the origin and the point  $x$  is decreasing with time, there is a positive contribution to the flux at the point  $x$ . This term does not appear in the standard Lagrange gradient model, but must be considered here due to the contour of the moving injection piston.

The numerous terms which appear in the model equations are all traceable to the velocity distribution, Equation (79). The role of the velocity distribution in the model equations is summarized in Table 1. The terms involving  $v(x,t)$  are related to the kinetic energy of the fluid, the acceleration of the flow, and in the momentum equation, the time rate of change of the average fluid momentum.

Gough<sup>10</sup> has presented a development of the ordinary differential equations of motion for piston motion and liquid injection, using the assumption that the liquid is incompressible. The resulting equations are similar in form to Equations

Table 1. Role of Pressure Distribution in the Model Equations.

Eq. No.	Equation	Terms Involving $v(x,t)$
Eq. (23)	Pressure Distribution	$\frac{1}{2}\rho v^2(x,t) \cdot \rho \int_0^x v(x',t) dx'$
Eq. (24)	Unsteady Bernoulli	$\rho \int_0^x \dot{v}(x,t) dx$
Eq. (27)	Space Mean Pressure	$\frac{1}{2}\rho \overline{v^2(t)} \cdot \rho \overline{vl(t)}$
Eq. (57)	Momentum	$\frac{1}{2} \{m_L \dot{v}(t)\}$

(64) and (65); however, Gough has approximated the integrals appearing in Equations (24) and (57) and used the assumption of incompressibility to further simplify the solution.

Comparing Gough's results with Equations (64) and (65), the first term on the left hand side of Equation (64) is just the acceleration of the liquid in the orifice. Gough has approximated this term by  $\beta l_H \ddot{v}$ , where in Gough's notation,  $l_H$  is the length of the orifice,  $\dot{v}$  is the time derivative of the average liquid velocity in the orifice and  $\beta$  is "a numerical factor close to unity".<sup>10</sup> The function  $U^2(t)$  is associated with the kinetic energy of the liquid approaching the orifice. The term in Equation (64) involving  $u_p$  would appear to be related to the acceleration of the liquid approaching the orifice; however, there is no comparable term in Gough's development.<sup>10</sup>

In Equation (65),  $M_p''(t)$  consists of the piston mass and the mass of liquid which is being accelerated in a region approaching the orifice. Similarly,  $m_L''(t)$  is associated with the mass of liquid being accelerated through the orifice. Following Gough's arguments, the last term on the right-hand side of Equation (65) would be related to "a correction to the pressure in the fuel chamber due to the velocity of the fuel,"<sup>10</sup> ie. the approach velocity of the LP.

## VII. SIMPLIFIED RLPG GEOMETRY

The coefficients arising in the model equations, Equations (66)-(71), are quite complex, and it is instructive to consider their values over the piston travel. In order to more easily evaluate these coefficients, it is useful to consider a simplified RLPG geometry. Therefore, we will consider a hypothetical variation of the concepts depicted in Figures 1 and 2, in which the bolt has a constant radius (i.e. no taper, front or rear) and the injection piston and transducer block are configured such that the reservoir and injection orifice are right circular cylindrical annuli. The functions describing the area and volume of the reservoir (analogous to Equations (4), (5) and (17) ) are

$$A(x,t) = A_L H(s_1 - x) + A_3 H(x - s_2) \quad (80)$$

$$a(x,t) = A_R H(x - s_2) \quad (81)$$

$$V(x,t) = A_L x H(s_1 - x) + [A_L l_1(t) + A_3(x - s_2)] H(x - s_2) \quad (82)$$

where  $l_1(t)$  is the distance from the transducer block to the point  $s_1$  on the injection piston.

Using Equations (80)-(82), the integrals required to evaluate the coefficients in the model equations are easily computed. We will focus on the effective lengths, Equations (66) and (67) and the effective masses, Equations (69) and (70), since they depend only on the geometry of the system and the piston position. The integrals required for this evaluation are,

$$L_1^{03}(t) = \frac{A_R l_2}{A_3} \quad (83)$$



$$L_2^{03}(t) = \frac{A_L}{V_R(t)} \left\{ \frac{1}{2} l_1(t)^2 + \frac{A_L}{A_3} l_1(t) l_2 + \frac{1}{2} l_2^2 \right\} \quad (84)$$

$$L_3^{03}(t) = \frac{1}{V_R(t)} \left\{ \frac{1}{2} A_L l_1(t)^2 + A_L l_1(t) l_2 + \frac{1}{2} A_3 l_2^2 \right\} \quad (85)$$

$$J_9^{03}(t) = \frac{A_R l_2}{V_R(t)} \quad (86)$$

where  $l_2 = s_3 - s_2$ , and

$$V_R(t) = A_L l_1(t) + A_3 l_2 \quad (87)$$

Parameters for our hypothetical gun configuration, based on an actual 30-mm RLPG, are presented in Table 2.

Table 2. 30-mm Gun Parameters

$l_1(0)$	7.4 cm
$l_2$	1.0 cm
$A_L$	25.2 cm <sup>2</sup>
$A_R$	23.3 cm <sup>2</sup>
$A_3$	1.9 cm <sup>2</sup>
$V_R(0)$	188.4 cm <sup>3</sup>
$m_L(0)$	269.8 gm
$\rho_L$	1.432 gm/cm <sup>3</sup>
$M_p$	2109.2 gm

These parameters have been used to evaluate Equations (83)-(89), which have subsequently been used to calculate the effective lengths and masses, Equations (66), (67), (69) and (70), also as a function of piston travel. The results are presented in Figures 4 and 5.

The behavior of  $L_u''(t)$  in Figure 4 is somewhat surprising. Its initial value is about 3.325 cm, or about 45% of the total piston travel. It decreases steadily, becoming negative at about 6.4 cm of piston travel, and has a final value of about -6.13 cm. Thus, the effect of piston acceleration over the first portion of travel is to increase the liquid acceleration  $\ddot{u}_3$ , while the effect is opposite over the last 1.0 cm of travel. In light of this sign change, it is difficult to attribute physical significance to this quantity.

The second effective length in Figure 4,  $L_v''(t)$ , has an initial value of about 1.27 cm, or about 1.27 times the orifice length. Its final value is 0.50 cm. At 6.4 cm travel, the value of  $L_v''(t)$  is still approximately 1.0 cm. The rapid drop to the final value occurs over the last 1.0 cm of piston travel. The values of this function over the first 6.4 cm of travel are consistent with the approximation used by Gough in reference 10.

In Figure 5, we see that the initial value of the effective piston mass,  $M_p''(t)$ , is about 5.8% higher than the actual piston mass. This represents about 122 g of liquid which is being accelerated in the region near the injection orifice, or about 45% of the total liquid mass at the beginning of the injection process. As the piston moves to the rear,  $M_p''(t)/M_p$  approaches a final value slightly less than 1.0.

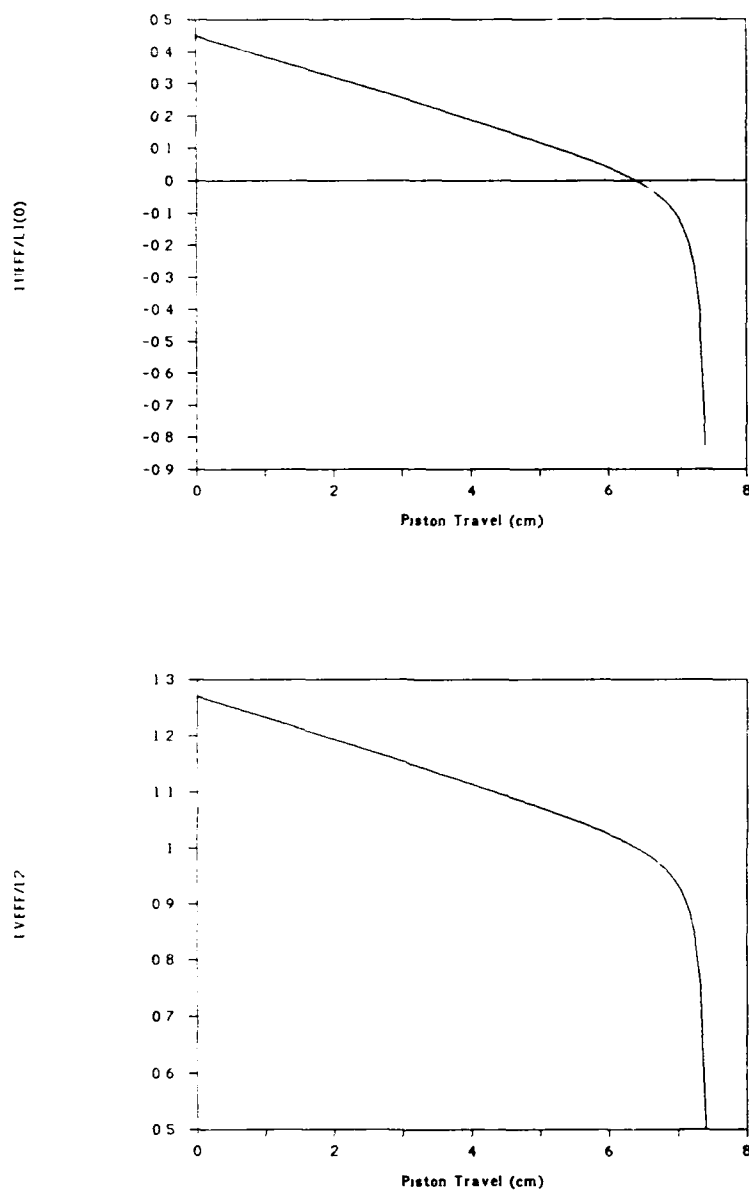


Figure 4. Normalized Effective Lengths,  $[L_e^{eff}(t)/L_1(0)]$  and  $[L_e^{eff}(t)/l_2]$  as Functions of Piston Travel for a Simplified RLPG Geometry.

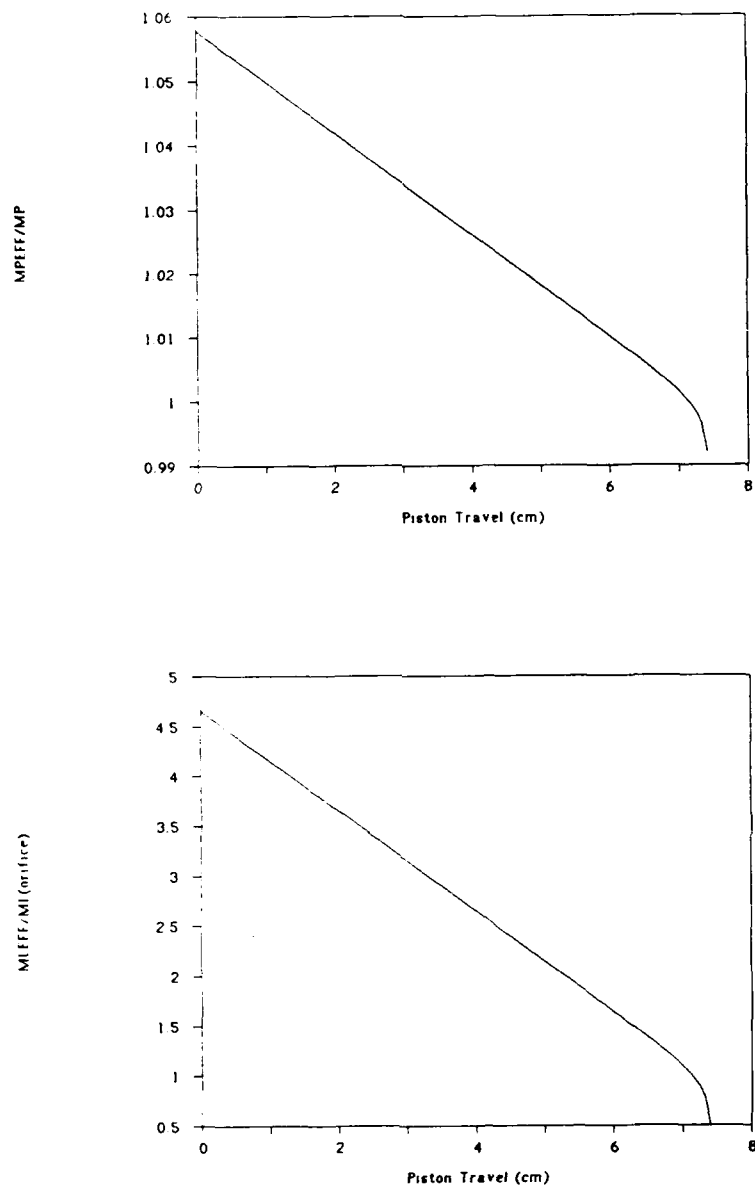


Figure 5. Normalized Effective Masses,  $[M_p^{eff}(t)/M_p]$  and  $[m_t^{eff}(t)/m_t^{orifice}]$  as Functions of Piston Travel for a Simplified RLPG Geometry.

The effective liquid mass,  $m_l^{eff}(t)$ , in Figure 5, has an initial value of about 12.67 g, or about 4.66 times the mass of the liquid in the orifice. As can be seen in Figure 5,  $m_l^{eff}(t)/m_l^{orifice}$  decreases steadily over the entire piston travel, reading a final value of 0.50.

The behaviors of these four coefficients are quite reasonable over most of the piston travel.  $L_l^{eff}(t)$  and  $m_l^{eff}(t)$  do begin to change rapidly very close to the end of piston stroke, and the effective length coefficient,  $L_l^{eff}(t)$ , becomes negative at a piston travel of 6.4 cm when the ratio of the reservoir area,  $A_L$ , to the reservoir volume begins to approach 1. It is important to remember that we have attempted to describe a flow process which is at least two-dimensional with a quasi one-dimensional model. Over most of the piston travel, while the reservoir length is large compared to its radial dimension, this approximation is not unreasonable. However, as the piston approaches the rear wall, the two-dimensional effects become dominant. The major component of the flow velocity will actually be in the radial direction near the end of piston travel. Therefore, we anticipate that the model for the flow in the LP reservoir and injection orifice which has been developed here, might not be valid near the end of the piston travel. However, as will be demonstrated in the following paper, the model accurately reproduces the experimental LP reservoir pressure, even in the latter stages of piston motion, indicative of the fact that the correction terms are small and the process is dominated by the piston inertia.

### VIII. CONCLUSION

A model of the LP reservoir and LP injection in a regenerative LP gun has been presented. This model is a revision of an earlier version, the primary difference being the inclusion of the full pressure distribution in the model equations.

This model is based on a generalization of the Lagrange approximation to address the variation of fluid mass in the reservoir during the ballistic cycle; the variation of area with position in the reservoir; and the variation of area with time at a fixed position in reservoir due to the rearward motion of the contoured injection piston.

In a companion report,<sup>9</sup> this model is applied to simulate the injection process of two RLPG configurations and the results are compared with experimental data.

## REFERENCES

1. Gough, P.S., "A Model of the Interior Ballistics of Hybrid Liquid Propellant Guns," BRL Contract Report No. BRL-CR-566, March 1987.
2. Gough, P.S., BRL Contract Report in preparation.
3. Coffee, T.P., "A Lumped Parameter Code for Regenerative Liquid Propellant Guns," BRL Technical Report No. BRL-TR-2703, December 1985.
4. Morrison, W.F. and Wren, G.P., "A Model of Liquid Injection in a Regenerative Liquid Propellant Gun," BRL Technical Report No. BRL-TR-2851, July 1987.
5. Wren, G.P. and Morrison, W.F., "Velocity and Pressure Distributions in the Liquid Reservoir in a Regenerative Liquid Propellant Gun," BRL Technical Report No. BRL-TR-2933, September 1988.
6. Coffee, T.P., Wren, G.P., and Morrison, W.F., "Interior Ballistic Modeling of Regenerative Liquid Propellant Guns," Proceedings of the Tenth International Symposium on Ballistics, October 1987.
7. Pate, R.A. and Schlermen, C.P., "Flow Properties of LGP 1846 at Reduced Temperatures," Proceedings of the 22nd JANNAF Combustion Meeting, October 1985.
8. Coffee, T.P., "The Analysis of Experimental Measurements on Liquid Regenerative Guns," BRL Technical Report No. BRL-TR-2731, May 1986.
9. Wren, G.P. and Morrison, W.F., "Extension of a Model of Liquid Injection in a Regenerative Liquid Propellant Gun Based Upon Comparison with Experimental Results," 25th JANNAF Combustion Meeting, October 1988. (BRL Technical Report in printing.)
10. Gough, P.S., "Review of GE Model of Regenerative Liquid Propellant Guns," Interim Report on work performed under Retainer Agreement F13-59-11651, April 1977.

# LIST OF SYMBOLS

$A(x, t)$	Cross Sectional Area of the Flow
$a(x, t)$	$\pi R_0^2 - \pi R^2(x, t)$
$A_L(t)$	$A_R + A_3(t)$
$A_\mu$	Cross Sectional Area of Chamber Side of Piston
$A_R$	Cross Sectional Area of Reservoir Side of Piston
$A_T$	Cross Sectional Area of Transducer Block
$A_3(t)$	Cross Sectional Area of Injection Orifice
$H(x)$	Heavyside Function
$J_n(x, t)$	Non-Dimensional Integral Functions Arising in Model Equations
$J_n^{03}(t)$	$J_n(s_3, t)$
$\overline{J_n^{03}(t)}$	Space Mean Value of $J_n(x, t)$
$L_n(x, t)$	Function with Units of Length Arising in Model Equations
$L_n^{03}(t)$	$L_n(s_3, t)$
$\overline{L_n^{03}(t)}$	Space Mean Value of $L_n(x, t)$



$L_u^{eff}(t)$	Effective Length Coefficient
$L_l^{eff}(t)$	Effective Length Coefficient
$l_1(t)$	$s_1 - s_0$
$l_2$	$s_3 - s_2$
$M_p$	Piston Mass
$M_p^{eff}(t)$	Effective Piston Mass
$m_l(t)$	Liquid Mass
$m_l^{orifice}$	Liquid Mass in Orifice
$m_l^{eff}(t)$	Effective Liquid Mass
$P(x,t)$	Liquid Pressure
$P_0(t)$	Liquid Pressure at Transducer Block
$P_3(t)$	Combustion Chamber Pressure
$\bar{P}(t)$	Space Mean Pressure in Liquid
$R(x,t)$	Radius of Inner Surface of Piston
$R_0$	$R(0,t)$

$r_b(x)$	Radius of Bolt
$s(x,t)$	Point on Inner Surface of Piston at Position $x$ at Time $t$
$U^2(t)$	Function with Units of Velocity Squared
$U^2 A(t)$	Function with Units of Velocity Squared times Area
$u_p$	Velocity of Piston
$V_R(t)$	Volume of Reservoir
$v(x,t)$	Liquid Velocity
$v_3$	Liquid Velocity at Orifice Exit
$\overline{v(t)}$	Space Mean Liquid Velocity
$\overline{v^2(t)}$	Space Mean Average of Square of Liquid Velocity
$\overline{vl(t)}$	Space Mean Average of $\int_0^x v(x',t) dx'$
$\rho$	Liquid Density

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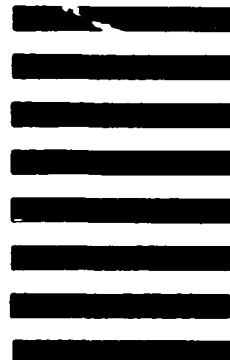
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